



IIT ROORKEE



NPTEL ONLINE
CERTIFICATION COURSE

QUANTITATIVE INVESTMENT MANAGEMENT

LECTURE 24

Modeling of Fixed Income Returns

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PRICING WITH SLOPING YIELD CURVE

- With a sloped yield curve that may not always be true in the shorter run (before maturity).
- If the yield curve is upward sloping, the rolldown return will be higher than the start-of-period YTM.
- For an upward sloping yield curve, the yields at the end of the holding period (based on the term to maturity less the investment horizon) will be lower than the $t=0$ yield which is based on a longer term to maturity.
- Thus, the bond value at the end of the holding period will be priced at a lower YTM than at $t=0$.



ILLUSTRATION

- Assume a 10% 2-year annual coupon bond (FV=1,000). Calculate its current price and price at the end of one year assuming the yield curve to be upward sloping with $S_{01} = 6.2160\%$, $S_{02} = 15.5000\%$. Assume that the rates at $t=1$ year for various maturities are exactly the same as at $t=0$.

SOLUTION

- We have: $P_0 = \frac{C_1}{(1+S_{01})} + \frac{C_2}{(1+S_{02})^2} = \frac{100}{(1+0.06216)} + \frac{1100}{(1+0.155)^2} = 918.72$
- Similarly, $P_1 = \frac{C_2}{(1+S_{01})} = \frac{1100}{1.06216} = 1035.63$
- $y_{01} = \frac{1035.63+100}{918.72} - 1 = 23.61\%$
- $y_{12} = \frac{1100-1035.63}{1035.63} = 6.21\%$

ILLUSTRATION

- Assume a 10% 2-year annual coupon bond (FV=1,000). Calculate its current price and forward price at the end of one year assuming the yield curve to be upward sloping with $S_{01} = 6.2160\%$, $S_{02} = 15.5000\%$.

SOLUTION

- We have: $P_0 = \frac{C_1}{(1+S_{01})} + \frac{C_2}{(1+S_{02})^2} = \frac{100}{(1+0.06216)} + \frac{1100}{(1+0.155)^2} = 918.72$
- Similarly, $P_1 = \frac{C_2}{(1+f_{12})} = \frac{C_2(1+S_{01})}{(1+S_{02})^2} = \frac{1100 \times 1.06216}{1.155^2} = 875.83$
- $y_{01} = \frac{875.83+100}{918.72} - 1 = 6.216\% = S_{01}$
- $y_{12} = \frac{1100-875.83}{875.83} = 25.60\% = f_{12}$

YIELD SPREADS



YIELD SPREADS

- A yield spread, in general, is the difference in yield between different fixed income securities.



YIELD SPREADS OVER BENCHMARK RATES

- In fixed-income security analysis, it is important to understand *why* bond prices and yields-to-maturity change.
- To do this, it is useful to separate a yield-to-maturity into two components: the benchmark and the spread.
- The benchmark yield for a fixed-income security with a given time-to-maturity is the base rate, often a government bond yield.
- The spread is the difference between the yield-to-maturity and the benchmark.



- The yield spread over a specific benchmark is referred to as the benchmark spread and is usually measured in basis points.
- If no benchmark exists for a specific bond's tenor or a bond has an unusual maturity, interpolation is used to derive an implied benchmark.
- Also, bonds with very long tenors are priced over the longest available benchmark bond.
- For example, 100-year bonds (often called “century bonds”) in the United States are priced over the 30-year US Treasury benchmark rate.

INTERPRETATION OF BENCHMARK RATES

- The reason for this separation is to distinguish between macroeconomic and microeconomic factors that affect the bond price and, therefore, its yield-to-maturity.
- The benchmark captures the macroeconomic factors: the expected rate of inflation in the currency in which the bond is denominated, general economic growth and the business cycle, foreign exchange rates, and the impact of monetary and fiscal policy.
- Changes in those factors impact all bonds in the market, and the effect is seen mostly in changes in the benchmark yield.



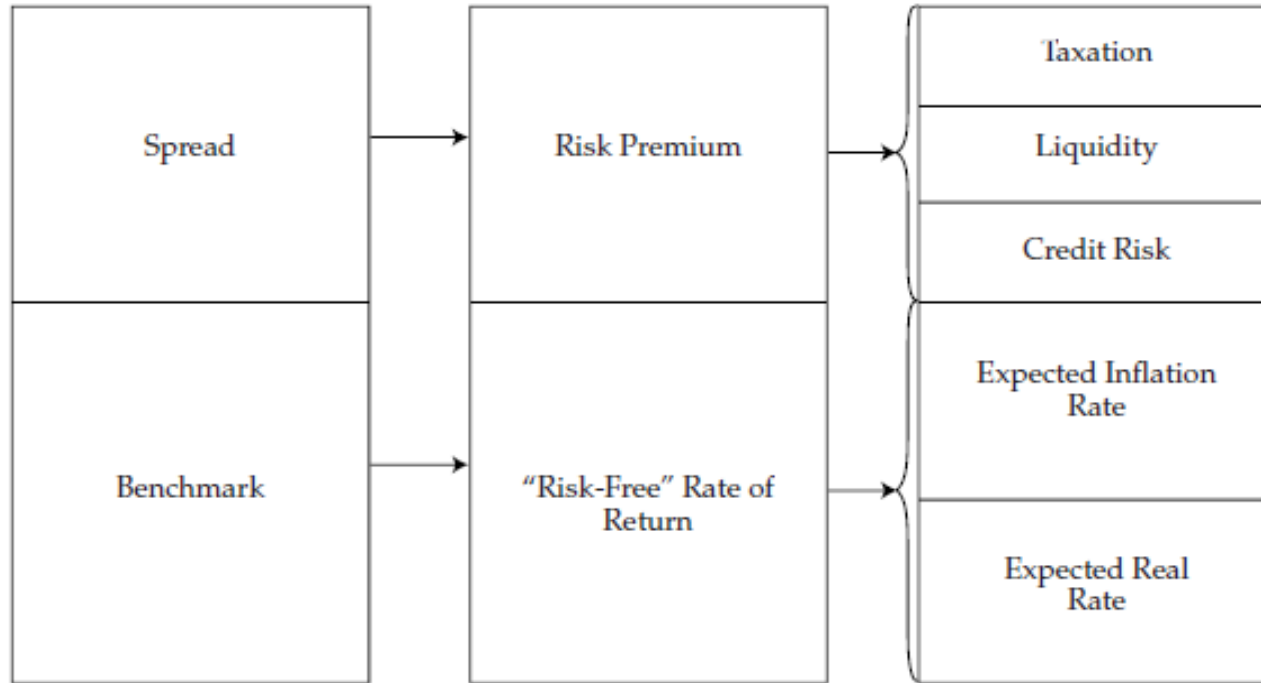
INTERPRETATION OF THE SPREAD

- The spread captures the microeconomic factors specific to the bond issuer and the bond itself: credit risk of the issuer and changes in the quality rating on the bond, liquidity and trading in comparable securities, and the tax status of the bond.
- It should be noted, however, that general yield spreads across issuers can widen and narrow with changes in macroeconomic factors.

COMPONENTS OF THE YIELD

- The benchmark is often called the risk-free rate of return.
- Also, the benchmark can be broken down into the expected real rate and the expected inflation rate in the economy.
- The yield spread is called the risk premium over the “risk-free” rate of return.
- The risk premium provides the investor with compensation for the credit and liquidity risks, and possibly the tax impact of holding a specific bond.

YTM BUILDING BLOCKS



- **The benchmark varies across financial markets. Fixed-rate bonds often use a government benchmark security with the same time-to-maturity as, or the closest time-to-maturity to, the specified bond.**
- **This benchmark is usually the most recently issued government bond and is called the on-the-run security.**

ON THE RUN TREASURIES

- The on-the-run government bond is the most actively traded security and has a coupon rate closest to the current market discount rate for that maturity. That implies that it is priced close to par value.
- Seasoned government bonds are called off the run
- On-the-run bonds typically trade at slightly lower yields-to-maturity than off -the-run bonds having the same or similar times-to-maturity because of differences in demand for the securities and, sometimes, differences in the cost of financing the government security in the repo market.

FIXED INCOME PORTFOLIO MANAGEMENT



MODELING FIXED INCOME RETURN



MODELING RETURN ON FIXED INCOME BONDS: ILLUSTRATION

- A fixed-income analyst wishes to forecast the expected return of a bond portfolio for the next year.
- He gathers the following information and assumes no reinvestment of coupon cash flow:
- **INCOME/CURRENT YIELD:**
- Par value (notional principal) in millions: 100
- Average coupon rate of portfolio: 3.0%
- Coupon frequency: Semiannual
- Horizon analysis: 1 year
- Average bond price of portfolio: 101.500

- **ROLLING YIELD:** Average life: 7 yrs
- YTM corresponding to price of 101.500: 2.76% p.a.
- Further assume the initial yield curve is upward sloping and the six-year bond yields 2.56%.
- A bond pricing model may be used to project the price of the seven year bond in one year when it is a six-year bond priced at a 2.56% yield. Assume that the projected price of portfolio in one year if yield curve is unchanged i.e. at a yield of 2.56% p.a.: 102.419

- **EXPECTED PRICE CHANGE BASED ON THE INVESTOR'S EXPECTED CHANGE IN YIELD AND SPREAD:**
- The analyst projects that a general decline in interest rates of **50 bps** is likely. A decline in credit spreads is expected as well.
- Combining the credit spread decline for the portion of the portfolio in credit-risky bonds with the general 50 bps decline, the analyst projects a 54 bps overall decline across the portfolio.
- Thus, Exp Yield & Spread change of portfolio (ΔY): (-) 0.54%
- Average modified convexity (C_{mod}) of portfolio: 28
- Average bond D_{mod} or MD of portfolio: 5.60

- **CREDIT LOSSES**

- The analyst estimates 0.071% of the portfolio's bond value will default and 15% of any defaulting value will be recovered, which means 85% of defaulting bond value will be lost.

- **EXPECTED GAINS OR LOSSES VERSUS INVESTOR'S CURRENCY**
- The portfolio is invested 40% in foreign denominated bonds, and the investor expects the foreign currencies (weight to reflect portfolio exposures) to appreciate 3.925%.

INCOME (CURRENT) YIELD

- Income (Current) Yield: Annual coupon payment/Current bond portfolio price $\frac{3.0}{101.50} = 2.956\%$.
- **Because this example assumes no cash-flow reinvestment, it does not matter that the coupons are paid semiannually.**
- With a 3.0% annual coupon rate, 1.50 will be received in 6 and 12 months for total coupons collected of 3.00 per 100 par.
- Collected coupon amount divided by initial price is the current income return and what this component measures.

6 MONTH PERIODIC RETURN

- This could be converted to a periodic return for periods other than a year.
- Over 6 months it would be Annual Income Yield/2
- $= 2.956/2 = 1.478\%$.

REINVESTMENT RETURN

- The average coupons were assumed to be paid semiannually with the first payment in 6 months and no reinvestment.
- If a 2% reinvestment rate of coupon payments is assumed, then $\frac{1}{2}$ of 3% will be collected in 6 months and reinvested for 6 more months at 1% periodic rate (2% reinvestment/2).
- That would provide another $1.5 \times 0.01 = 0.015\%$ of return.

ROLLDOWN YIELD

- **Rolldown yield relates to a bond pricing model assuming the yield curve is unchanged.**
- The portfolio is made up only of say a seven-year bond priced to yield 2.76% with a price of 101.50.
- Further given that the initial yield curve is upward sloping and the six-year bond yields 2.56%.
- Also given that the bond price one year from now i.e. as a 6-year bond with a YTM 2.56% p.a. is 102.419

- (Note that this is not necessarily a simple analysis for a portfolio of bonds and would have to be done bond by bond and then aggregated. That is why it is a given value in the data provided).
- The rolldown return is the bond's:
$$\frac{(\text{end of horizon period projected price} - \text{beginning price})}{\text{beginning price}}$$
- $$= \frac{(102.419 - 101.50)}{101.50} = 0.905\%$$

ROLLING YIELD

- **Components 1 and 2 (Income Yield & Rolldown Yield) are sometimes combined and called the Rolling Yield: $2.956 + 0.905 = 3.86\%$.**



- **Note that if the yield curve is flat and a bond is initially priced at a premium (discount) to par, the projected price at end of period will be lower (higher) than start-of-period price as the bond's price is pulled to par at expiration.**



- With a sloped yield curve that may not always be true in the shorter run (before maturity).
- If the yield curve is upward sloping, the rolldown return will be higher than the start-of-period YTM
- This is because the bond price P_H at the end of the holding period ($t=0$ to $t=H$) will be worked out at a lesser term to maturity ($t=H$ to $t=T$ i.e. $T-H$) than its price at $t=0$ which is calculated with a term to maturity $t=T$.
- If the curve is upward sloping, the shorter end rates are lower. Thus, the rate for maturity of $T-H$ are lower than those for maturity T . Hence, the price calculated at exit at $t=H$ will be at a lower rate & hence, higher.

EXPECTED PRICE CHANGE BASED ON THE INVESTOR'S EXPECTED CHANGE IN YIELD AND SPREAD:

- It is also projected that a general decline in interest rates of 50 basis points is likely. A decline in credit spreads is expected as well.
- Weighting the credit spread decline for the portion of the portfolio in credit-risky bonds with the general 50 basis point decline, he projects a 54 basis point overall decline versus the 2.56% yield used in the rolldown calculation.
- Thus, expected average yield and spread change of portfolio (ΔY):
-0.54%
- Average bond convexity (C) of portfolio: 28
- Average bond D_{mod} or MD of portfolio: 5.60

SOLUTION

- Expected price change is calculated from the investor's expected change in yield and spreads using the portfolio's duration and convexity:
- $(-MD \times \Delta Y) + (\frac{1}{2}C \times \Delta Y^2)$
- $= (-5.6 \times -0.0054) + (\frac{1}{2} \times 28 \times 0.0054^2)$
- $= 0.03024 + 0.00041 = 3.065\%$

CREDIT LOSSES

- The analyst estimates 0.071% of the portfolio's bond value will default and 15% of any defaulting value will be recovered, which means 85% of defaulting bond value will be lost.
- Thus, The expected credit losses $0.071\% \times 0.85 = 0.060\%$.

EXPECTED GAINS OR LOSSES VERSUS INVESTOR'S CURRENCY

- The portfolio is invested 40% in foreign denominated bonds, and the investor expects the foreign currencies (weight to reflect portfolio exposures) to appreciate 3.925%, giving his portfolio an expected gain of $3.925\% \times 0.40 = 1.570\%$.

RETURN COMPONENTS: SUMMARY

1. Yield income: Annual coupon/current bond price:
 $3.0/101.50 = 2.956\%$
2. Rolldown return: (projected ending bond price (BP) – beginning BP)/ beginning BP; based on no change in the yield curve $(102.419 - 101.50)/101.50 = 0.905\%$
3. Price change due to investor yield change predictions: $(-MD \times \Delta Y) + (\frac{1}{2}C \times \Delta Y^2) = (-5.6 \times (-0.0054)) + (\frac{1}{2} \times 28 \times 0.00542) = 3.065\%$

- 1. Credit losses: predicted default adjusted for recovery rate: $0.071\% \times 0.85 = -0.060\%$**
- 2. Currency G/L: projected change in value of foreign currencies weighted for exposure to the currency: $3.925\% \times 0.40 = 1.570\%$**
- 3. Expected total return for the year = 8.395%**